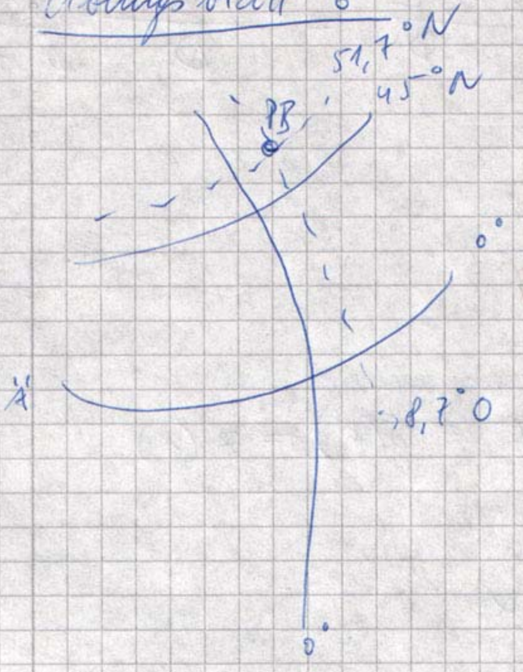
Übungsblatt 6 (25.05.05)

a)



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} r \cos\varphi \cos\theta \\ r \sin\varphi \cos\theta \\ r \sin\theta \end{pmatrix}$$

$$\vec{PB} = \begin{pmatrix} 6370 \cos(8{,}7) \cos(51{,}7) \\ 6370 \sin(8{,}7) \cos(51{,}7) \\ 6370 \sin(51{,}7) \end{pmatrix} = \begin{pmatrix} 3902{,}57 \\ 597{,}18 \\ 4999{,}03 \end{pmatrix}$$

$$\vec{s_1} = \begin{pmatrix} 25000 \\ 0 \\ 0 \end{pmatrix} \qquad \vec{s_2} = \begin{pmatrix} 0 \\ 0 \\ 25000 \end{pmatrix} \qquad \vec{s_3} = \begin{pmatrix} 12500 \\ 12500 \\ 17677{,}67 \end{pmatrix}$$

$$|\vec{PB\,s_1}| = 21689{,}82\,km \Rightarrow t_1 = \frac{|\vec{PB\,s_1}|}{c} = 0{,}072\,s$$

$$|\vec{PB\,s_2}| = 20386{,}99\,km \Rightarrow t_2 = 0{,}068\,s$$

$$|\vec{PB\,s_3}| = 19399{,}51\,km \Rightarrow t_3 = 0{,}065\,s$$

A2

a) $F = -m a_z = \underbrace{-mg}_{\text{Gravitationskraft}} - M \frac{\partial}{\partial z} \vec{H}_A = -mg - M H_{Az}$

$\Rightarrow$ Schiefer Wurf (Bahn)

$$v_{x_0} = \frac{x}{t} \Rightarrow t = \frac{x}{v_{x_0}}$$

$$z(t) = -\frac{1}{2} a_z t^2 + v_{z_0} t$$

$$z(L) = -\frac{1}{2} a_z \left(\frac{L}{v_{x_0}}\right)^2 + v_{z_0} \frac{L}{v_{x_0}}$$

$$\tan\alpha = \frac{v_{z_0}}{v_{x_0}}$$

$$\tan\beta = \frac{v_z(L)}{v_{x_0}} = \frac{z(L)}{D/2}$$

$$\tan\beta = \frac{v_z(L)}{v_{x_0}} = \left(-a \frac{L}{v_{x_0}} + v_{z_0}\right) \frac{1}{v_{x_0}} = \tan\alpha - \frac{aL}{v_{x_0}^2}$$

$$\tan\beta = \frac{2z(L)}{D} = -\frac{aL^2}{D a_{x_0}} + \frac{2v_{z_0}L}{v_{x_0}D} = 2\frac{L\tan\alpha}{D} - \frac{aL^2}{D v_{x_0}^2}$$

$$\tan\alpha = \frac{a}{v_{x_0}^2}\ \frac{DL-L^2}{D-2L}$$

b) $y(x) = mx^2 + bx + c$

Steigung: $y'(x) = 2mx + b = v(x) = z'(x)$

$z'(0) = b = \tan\alpha$

$z(x) = -\frac{1}{2}a_z \frac{x^2}{v_{x_0}^2} + x\tan\alpha$

$z'(x) = -a_z \frac{x}{v_{x_0}^2} + \tan\alpha$

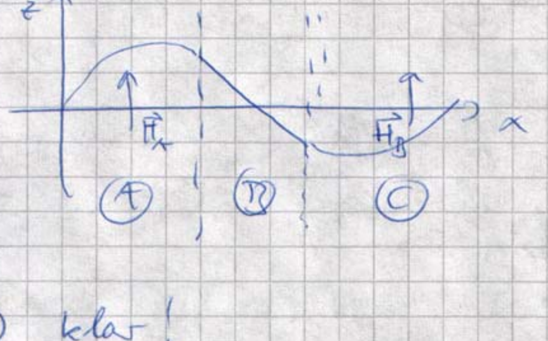
Steigung $z(L)$ $\tan\beta = \frac{2z(L)}{D} = z'(L)$

$$z'(L) = \frac{2z(L)}{D} = 2mL + \tan\alpha$$

$$\Rightarrow m = \frac{2z(L) - D\tan\alpha}{2DL} = \frac{L + \frac{D}{2}}{L(L+D)}\tan\alpha$$

$$\frac{dz}{dx} = 0 \Rightarrow x = \frac{L(L+D)}{2\left(L+\frac{D}{2}\right)} = 0{,}8\,m$$

$z(0{,}8) = 0{,}035\,m$



c) klar!